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TWO-DIMENSIONAL MESH ADJUSTMENT ALGORITHM
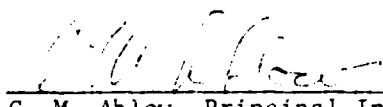
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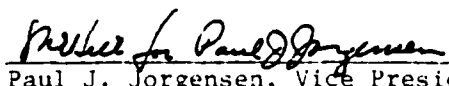   Program Manager

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During this third year of research the particular, previously presented, mesh adjustment algorithms for the finite difference solution of ordinary differential equation systems have been shown to be instances of the use of monitor functions intrinsic to the system and its solution. These monitors have been found to be nearly as accurate as the monitor that minimizes the truncation error. The weighted normalizing monitor, which uses the sum of the squares of derivatives of the solution function, has been applied in a two-dimensional mesh adjustment algorithm.

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I RESEARCH TASKS

The general objective of this research is to establish a transformation of coordinates that facilitates the finite difference solution of partial differential equations in two dimensions. The objective can be attained by completion of the following tasks.

A. Coupled Systems of Ordinary Differential Equations

Two-dimensional mesh adjustment is obtained from an appropriately coupled system of one-dimensional boundary-value problems. The one-dimensional algorithm has been tested on single differential equations of low order. We must verify that the method is successful with coupled systems. A coupled system of this kind is provided by the equations for conservation of mass and energy in a steadily fed chemical reaction.

B. Known Boundary Layer

An example in which the mesh adjustment can be well approximated in advance is provided by a partial differential equation with a solution having a boundary layer in known position. Difficulties in such examples concern the adequacy of the difference scheme to couple the one-dimensional problems on the coordinate lines transverse to the boundary layer. Problems of viscous flow past obstacles are of this type.

C. Unknown Boundary Layer

The exceptional size of a parameter in a problem generally causes the sharp variations in the boundary layer. Continuation of the solution from the readily calculated smooth variations for a parameter of ordinary size is successful in one dimension. Continuation will apply in two dimensions provided that a way is found to ensure that the coordinate lines transverse to the developing boundary layer are identified early in the continuation process. Internal boundary layers whose location is determined by the solution itself arise, for example, in flow-through chemical reactors and field-effect transistors.

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II STATUS OF THE RESEARCH EFFORT

During the past year the algorithm for automatic mesh adjustment
by transformation to a campylotropic coordinate[1]* that depends on the
length and curvature of the solution curve has been shown to be in the
general class of algorithms based on monitor functions--that is, on
functions that take equally spaced values at mesh points. Intrinsic
monitor functions that depend on the solution itself are particularly
useful for mesh adjustment. We have compared[2] several functions of this
kind by application to a typical second-order ordinary differential
equation. The normalizing monitor, based on a weighted sum of the squares
of the derivatives of the dependent variables, and the campylotropic
coordinate were found to give difference approximations with much greater
accuracy for the same number of mesh points than a uniform mesh does.
The accuracy was similar to that of the optimal monitor that minimizes
the truncation error.

A.   Coupled Systems of Ordinary Differential Equations

The normalizing monitor function has immediate application to
systems, while the other intrinsic monitors require extension and
generalization. The normalizing monitor has been shown[2] to be effective
on the example of three, coupled nonlinear, second-order, ordinary
differential equations that model the steady decomposition of ozone in
a flow-through reactor. A curvature-dependent monitor requires a more
intricate computation than does the normalizing monitor, and the results
are not necessarily more accurate. The optimal monitor involves the
fifth derivatives of the functions appearing in the equations and is
therefore probably too complex for general use.

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*Numbers denote papers in the Publication List, Section III.

B. **Known Boundary Layer**

See discussion for Task C.

C. **Unknown Boundary Layer**

Tasks B and C for partial differential systems in two dimensions
have been considered together because, as with the one-dimensional
algorithms, no a priori knowledge of the location of the boundary layers
is used in the mesh adjustment process. The mesh adjustment is made by a
transformation of coordinates to a solution-dependent monitor function
for one independent variable and an orthogonal coordinate for the second
variable. The method has been applied[3] to a boundary value problem for
Poisson's equation with an analytic solution having a sharp variation
along a chord of a circular disc. An initially arbitrary coordinate
is iteratively improved under the normalizing monitor requirement to
an orientation with one coordinate along the chord and the desired small
spacing in the other coordinate across the chord. Application to flows
over airfoils with viscous boundary layers or detached shocks is now
under way.

III PUBLICATIONS LIST

1. C. M. Ablow and S. Schechter, "Campylotropic Coordinates," Compu-
    tational Physics (June 1978).
2. C. M. Ablow, S. Schechter, and W. E. Zwisler, "Node Selection for
    Two-Boundary Value Problems," submitted to Computational Physics.
3. C. M. Ablow and S. Schechter, "Generation of Boundary and Boundary-
    Layer Fitting Grids," in preparation. An abstract is attached.

IV PERSONNEL

Mr. W. E. Zwisler joined the co-investigators, Drs. C. M. Ablow
and S. Schechter, in the research effort.

V INTERACTIONS

The contents of the second paper in the publications list were presented at the fall meeting of S.I.A.M. in Denver, November 12-14, 1979.

VI SPECIFIC APPLICATIONS

The ozone decomposition example presented in the second publication is an application of the method to a chemical process in a flow-through reactor. Direct application to jet or rocket engines in one-dimensional steady-state approximation should be possible.

Generation of Boundary and Boundary-Layer Fitting Grids

by

C. M. Ablow and S. Schechter
SRI International

ABSTRACT

A grid that improves the accuracy and speed of computation with a
given finite difference approximation to a boundary value problem for
a differential equation is more satisfactory than other grids. A
best method of grid generation will therefore depend on the problem
domain, the solution, and the difference scheme.

An automatic generator for the grid that minimizes the truncation
error of a given difference scheme for two-point boundary value problems
over a finite one-dimensional interval has been previously presented.[*]
This truncation error minimizing (TEM) generator changes the independent
variable to one in which uniformly spaced nodes fit the boundaries and
cluster in any boundary layers where the solution has a sharp variation.
The number of nodes and the complexity of the calculation are known
in advance so that the time and cost of the calculation can be estimated.
Other generators producing grids that equally distribute measures of the
solution curve arc length or length and curvature were found to be about
as accurate as the TEM generator but more easily implemented. The arc
length coordinate can also be defined as the transformation that minimizes
the sum of the squares of the derivatives of the dependent and independent
variables, a definition that readily generalizes to higher dimensions.

Experience with two-dimensional grid generation, as applied to a
Dirichlet problem for the Poisson equation on the unit disc, is presented.
The example has an analytic solution with sharp variation across a

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[*]C. M. Ablow, S. Schechter, and W. H. Zwisler, "Node Selection for Two-
Boundary Value Problems," sumbitted to Computational Physics.

diameter of the disc. The grid is uniformly rectangular on the unit square in the transformed coordinates. Transformations were chosen to minimize the sum of the squares of the derivatives of the dependent variable and of the dependent and originally independent variables. The TEM transformation was judged too complex to be practical. The results show that the grid fits the boundaries, clusters about the boundary layer, and rotates into alignment with it as desired.
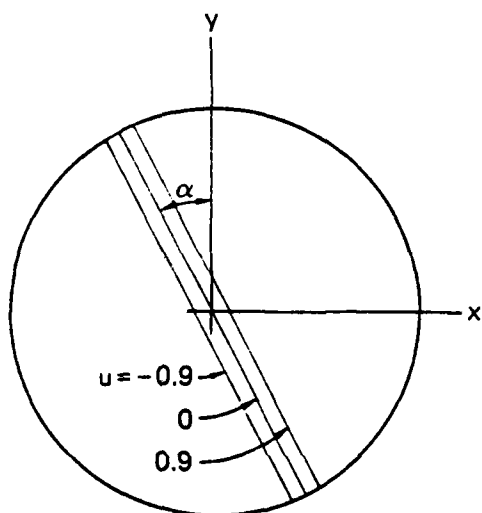
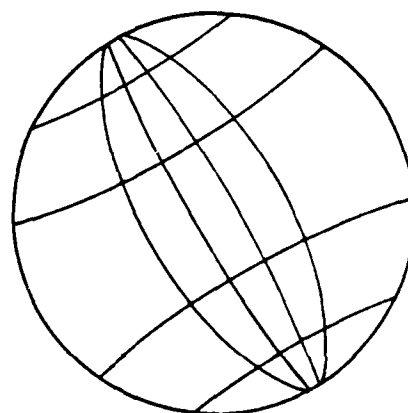$$u_{xx} + u_{yy} = 2p^2 \operatorname{sech}^2 pz \tanh pz$$

$$u = \tanh pz$$

$$z = x \cos \alpha + y \sin \alpha$$

$$p = 20$$

$$a = \pi/6$$

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FIGURE 1   PROBLEM DOMAIN

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FIGURE 2   SOLUTION GRID (SKETCH)